6.3. Relations: Formal Semantics

1. Extensions of Relation Letters. When providing a semantic interpretation for predicate letters in the previous chapter, we began with a domain of discourse populated by one or more objects, and then provided each predicate letter in the model with a (possibly empty) set of those objects to 'apply' to.

$\mathbb{D}$: {**Neko**, **Lucretia**}

A: **Neko** **B**: **Lucretia**
C: **Jack**

$\mathbf{G^1}$: {**Lucretia**} $\mathbf{I^1}$: {**Neko**, **Lucretia**}
$\mathbf{H^1}$: {**Neko**, **Jack**} $\mathbf{J^1}$: { }

So if "G[1]" stands for "is human", "H[1]" for "is a cat", "I[1]" for "is female" and "J[1]" for "is a unicorn", then this model is a situation where Lucretia's human, Lucretia and Neko are female, Neko and Jack are cats, and nobody is a unicorn. We could think of the extension of a predicate as all the **'cases'** of that feature: all the examples (in this situation) of human-ness, of cat-hood, of womanicity, and of unicorn-ness.

That suggests a natural adaptation of the semantics to relations: the extension of a relation will again be just the cases of that relation appearing in a given situation. So for example, the extension of "likes" will be just the cases of liking occurring in that context. If in a certain situation Neko likes Jack and Jack likes Lucretia, there are two cases of liking going: one involving Neko and Jack, one involving Jack and Lucretia.

We might consider adding that to the model as follows (translating "likes" as "K[2]").

$\mathbf{K^2}$: {(**Neko**, **Jack**), (**Jack**, **Lucretia**)}

But keep in mind a point made earlier: it may be true in a situation that "Neko likes Jack" but false in that situation that "Jack likes Neko". So those two sentences

report **two different cases** of liking, which can't be slopped together in the semantics (on pain of confusing true sentences with false ones).

To emphasize the importance of who's doing what in a certain case, we use **ordered pairs** in the semantics to depict a case of a relation – marked as ordered by using angle brackets "<" and ">". So just as "Neko likes Jack" and "Jack likes Neko" make different claims, the following two ordered pairs are **different** cases of liking – the first where Neko likes Jack, the second where Jack likes Neko.

K²: {**<Neko**, **Jack>**, **<Jack**, **Neko>**}

Likewise with a three-place relation: it's one thing for Trixie to give Old Reliable to Trixie, and something else entirely for Trixie to give Old Reliable to Kitty. In the following model, with only one **ordered triple** in the extension of "L³," Kitty gave Old Reliable to Trixie and nobody gave anything else to anybody.

L³abc: **a gave b to c**

𝔻: {**Kitty**, **Old Reliable**, **Trixie**}

D: **Kitty** **E**: **Old Reliable**
F: **Trixie**

L³: {**<Kitty, Old Reliable, Trixie>**}

And in general: the semantic value of an n-place relation letter, in a particular model, is the **extension of** that ***n*-place relation letter**: a (possibly empty) **set of ordered *n*-tuples of objects** from the domain of that model (so: order pairs for a 2-place relation letter, ordered triples for a 3-place relation letter, and so on).[1]

> **The semantic value of an *n*-place relation letter in model *M* is a set of zero or more ordered *n*-tuples of objects drawn from the domain of *M*.**

[1] Consistent with this, the extension of a **1-place relation letter** – that is, a **predicate letter** – is a (possibly empty) set of ordered *1*-tuples of objects from the domain of the model in question. But a 1-tuple of objects is just: an object. (There's no difference between a one-object series or chain, and that object.) Hence the extension of a predicate letter is a (possibly empty) set of objects from the domain of the model – just as in Chapter Five.

An ordered n-tuple of objects can have the same object show up more than once. For instance, the following is an ordered pair where Neko holds both positions.

<Neko, Neko>

And a good thing, too: for in a situation where Neko likes herself, Neko is both the 'liker' and the 'like-ee'. Hence a model where the sentence "Neko likes herself" is true will be a model where the above ordered pair shows up in the extension of "likes" (or rather: in the extension of the formal translation of "likes" – say, "L^2").

2. A Shorthand Notation for Models.

[Then: dots and arrows shorthand]